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ON THE AUTOMORPHISM GROUP OF A HRUSHOVSKI'S PSEUDOPLANE ASSOCIATED TO $5/8$

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ABSTRACT. The automorphism group of Hrushovski's pseudoplane associated to $5/8$ is a simple group.

Hrushovski's construction, automorphism group, simple group
03C10, 03C13, 03C25, 03C30

1. INTRODUCTION

D. Evans, Z. Ghadernezhad, and K. Tent have shown that the automorphisms groups of certain countable structures obtained using the Hrushovski amalgamation method are simple groups. Among them, there are generic structure of \mathbf{K}_f for certain f with coefficient $1/2$ for the predimension function. They conjectured that the automorphism group of the generic structure of \mathbf{K}_f is a simple group if the coefficient of the predimension function for \mathbf{K}_f is rational.

In this paper, we show that the automorphism group of Hrushovski's original pseudoplane associated to a predimension function with coefficient $5/8$ is a simple group. Actually, we prove a sufficient condition given by Evans, Ghadernezhad, and Tent. We are going to treat the general rational cases in another paper [14].

We essentially use notation and terminology from Baldwin-Shi [3] and Wagner [15]. We also use some terminology from graph theory [4].

For a set X , $[X]^n$ denotes the set of all subsets of X of size n , and $|X|$ the cardinality of X .

We recall some of the basic notions in graph theory we use in this paper. These appear in [4]. Let G be a graph. $V(G)$ denotes the set of vertices of G and $E(G)$ the set of edges of G . $E(G)$ is a subset of $[V(G)]^2$. $|G|$ denotes $|V(G)|$. The *degree* of a vertex v is the number of edges at v . A vertex

of degree 1 is a *leaf*. G is a *path* $x_0x_1 \dots x_k$ if $V(G) = \{x_0, x_1, \dots, x_k\}$ and $E(G) = \{x_0x_1, x_1x_2, \dots, x_{k-1}x_k\}$ where the x_i are all distinct. x_0 and x_k are *ends* of G . The number of edges of a path is its *length*. A path of length 0 is a single vertex. G is a *cycle* $x_0x_1 \dots x_{k-1}x_0$ if $k \geq 3$, $V(G) = \{x_0, x_1, \dots, x_{k-1}\}$ and $E(G) = \{x_0x_1, x_1x_2, \dots, x_{k-2}x_{k-1}, x_{k-1}x_0\}$ where the x_i are all distinct. The number of edges of a cycle is its *length*. A non-empty graph G is *connected* if any two of its vertices are linked by a path in G . A *connected component* of a graph G is a maximal connected subgraph of G . A *forest* is a graph not containing any cycles. A *tree* is a connected forest.

To see a graph G as a structure in the model theoretic sense, it is a structure in language $\{E\}$ where E is a binary relation symbol. $V(G)$ will be the universe, and $E(G)$ will be the interpretation of E . The language $\{E\}$ will be called *the graph language*.

Suppose A is a graph. If $X \subseteq V(A)$, $A|X$ denotes the substructure B of A such that $V(B) = X$. If there is no ambiguity, X denotes $A|X$. We usually follow this convention. $B \subseteq A$ means that B is a substructure of A . A substructure of a graph is an induced subgraph in graph theory. $A|X$ is the same as $A[X]$ in Diestel's book [4].

We say that X is *connected* in A if X is a connected graph in the graph theoretical sense [4]. A maximal connected substructure of A is a *connected component* of A .

Let A, B, C be graphs such that $A \subseteq C$ and $B \subseteq C$. AB denotes $C|(V(A) \cup V(B))$, $A \cap B$ denotes $C|(V(A) \cap V(B))$, and $A - B$ denotes $C|(V(A) - V(B))$. If $A \cap B = \emptyset$, $E(A, B)$ denotes the set of edges xy such that $x \in A$ and $y \in B$. We put $e(A, B) = |E(A, B)|$. $E(A, B)$ and $e(A, B)$ depend on the graph in which we are working.

Let D be a graph and A, B , and C substructures of D . We write $D = B \otimes_A C$ if $D = BC$, $B \cap C = A$, and $E(D) = E(B) \cup E(C)$. $E(D) = E(B) \cup E(C)$ means that there are no edges between $B - A$ and $C - A$. D is called a *free amalgam* of B and C over A . If A is empty, we write $D = B \otimes C$, and D is also called a *free amalgam* of B and C .

Definition 1.1. Let α be a real number such that $0 < \alpha < 1$.

- (1) For a finite graph A , we define a predimension function δ by $\delta(A) = |A| - \alpha|E(A)|$.
- (2) Let A and B be substructures of a common graph. Put $\delta(A/B) = \delta(AB) - \delta(B)$.

Definition 1.2. Let A and B be graphs with $A \subseteq B$, and suppose A is finite.

$A \leq B$ if whenever $A \subseteq X \subseteq B$ with X finite then $\delta(A) \leq \delta(X)$.

$A < B$ if whenever $A \subsetneq X \subseteq B$ with X finite then $\delta(A) < \delta(X)$.

We say that A is *closed* in B if $A < B$.

$A <^- B$ if whenever $A \subsetneq X \subsetneq B$ with X finite then $\delta(A) < \delta(X)$.

Let \mathbf{K}_α be the class of all finite graphs A such that $\emptyset < A$.

Some facts about $<$ appear in [3, 15, 16]. Some proofs are given in [12].

Fact 1.3. If $A < B \subseteq D$ and $C \subseteq D$ then $A \cap C < B \cap C$.

Fact 1.4. Let $D = B \otimes_A C$.

- (1) $\delta(D/A) = \delta(B/A) + \delta(C/A)$.
- (2) If $A < C$ then $B < D$.
- (3) If $A < B$ and $A < C$ then $A < D$.

Fact 1.5. (1) Let A, B, C and D be graphs with $D = B \otimes C$ and $A \subseteq D$. Then $\delta(D/A) = \delta(B/A \cap B) + \delta(C/A \cap C)$.

- (2) Let D be a graph and A a substructure of D . Let $\{D_1, D_2, \dots, D_k\}$ be the set of all connected components of D where the D_i are all distinct. Then

$$\delta(D/A) = \sum_{i=1}^k \delta(D_i/A \cap D_i).$$

Let B, C be graphs and $g : B \rightarrow C$ a graph embedding. g is a *closed embedding* of B into C if $g(B) < C$. Let A be a graph with $A \subseteq B$ and $A \subseteq C$. g is a *closed embedding over A* if g is a closed embedding and $g(x) = x$ for any $x \in A$.

In the rest of the paper, \mathbf{K} denotes a class of finite graphs closed under isomorphisms.

Definition 1.6. Let \mathbf{K} be a subclass of \mathbf{K}_α . $(\mathbf{K}, <)$ has the *amalgamation property* if for any finite graphs $A, B, C \in \mathbf{K}$, whenever $g_1 : A \rightarrow B$ and $g_2 : A \rightarrow C$ are closed embeddings then there is a graph $D \in \mathbf{K}$ and closed embeddings $h_1 : B \rightarrow D$ and $g_2 : C \rightarrow D$ such that $h_1 \circ g_1 = h_2 \circ g_2$.

\mathbf{K} has the *hereditary property* if for any finite graphs A, B , whenever $A \subseteq B \in \mathbf{K}$ then $A \in \mathbf{K}$.

\mathbf{K} is an *amalgamation class* if $\emptyset \in \mathbf{K}$ and \mathbf{K} has the hereditary property and the amalgamation property.

A countable graph M is a *generic structure* of $(\mathbf{K}, <)$ if the following conditions are satisfied:

- (1) If $A \subseteq M$ and A is finite then there exists a finite graph $B \subseteq M$ such that $A \subseteq B < M$.
- (2) If $A \subseteq M$ then $A \in \mathbf{K}$.
- (3) For any $A, B \in \mathbf{K}$, if $A < M$ and $A < B$ then there is a closed embedding of B into M over A .

Let A be a finite structure of M . There is a smallest B satisfying $A \subseteq B < M$, written $\text{cl}(A)$. The set $\text{cl}(A)$ is called the *closure* of A in M .

Fact 1.7 ([3, 15, 16]). *Let $(\mathbf{K}, <)$ be an amalgamation class. Then there is a generic structure of $(\mathbf{K}, <)$. Let M be a generic structure of $(\mathbf{K}, <)$. Then any isomorphism between finite closed substructures of M can be extended to an automorphism of M .*

Definition 1.8. Let \mathbf{K} be a subclass of \mathbf{K}_α . $(\mathbf{K}, <)$ has the *free amalgamation property* if whenever $D = B \otimes_A C$ with $B, C \in \mathbf{K}$, $A < B$ and $A < C$ then $D \in \mathbf{K}$.

By Fact 1.4 (2), we have the following.

Fact 1.9. *Let \mathbf{K} be a subclass of \mathbf{K}_α . If $(\mathbf{K}, <)$ has the free amalgamation property then it has the amalgamation property.*

Definition 1.10. Let \mathbb{R}^+ be the set of non-negative real numbers. Suppose $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a strictly increasing concave (convex upward) unbounded function. Assume that $f(0) = 0$, and $f(1) \leq 1$. We assume that f is piecewise smooth. $f'_+(x)$ denotes the right-hand derivative at x . We have $f(x+h) \leq f(x) + f'_+(x)h$ for $h > 0$. Define \mathbf{K}_f as follows:

$$\mathbf{K}_f = \{A \in \mathbf{K}_\alpha \mid B \subseteq A \Rightarrow \delta(B) \geq f(|B|)\}.$$

Note that if \mathbf{K}_f is an amalgamation class then the generic structure of $(\mathbf{K}_f, <)$ has a countably categorical theory [16].

A graph X is *normal to f* if $\delta(X) \geq f(|X|)$. A graph A belongs to \mathbf{K}_f if and only if U is normal to f for any substructure U of A .

2. THEOREMS BY EVANS, GHADERNEZHAD, AND TENT

In this section, we fix a generic structure M of \mathbf{K}_f . Many of the following definitions and facts are by Evans, Ghadernezhad, and Tent[5].

Definition 2.1. Let $A \subseteq M$. $\text{Aut}(M/A)$ denotes the set of automorphisms of M fixing A pointwise. Let $b \in M$. $\text{orb}(b/A)$ denotes the $\text{Aut}(M/A)$ -orbit of b . So, $\text{orb}(b/A) = \{\sigma(b) \mid \sigma \in \text{Aut}(M/A)\}$.

Definition 2.2. Let $A \subseteq M$ be finite. The *dimension* $d(A)$ of A is defined by $d(A) = \delta(\text{cl}(A))$. Let $B \subseteq M$ be also finite. The *relative dimension* $d(A/B)$ is defined by $d(A/B) = d(AB) - d(B)$.

Definition 2.3. Suppose $b \in M$ and $A < M$ with A finite. We say that b is *basic* over A if $b \notin A$ and whenever $A \subseteq C < M$ and $d(b/C) < d(b/A)$ then $b \in C$. In this case, $\text{orb}(b/A)$ is called a *basic orbit* over A .

Definition 2.4. We say that M is *monodimensional* if for every finite $A < M$ and basic orbit D over A there is a finite $B < M$ with $M = \text{cl}(BD)$ and $A \subseteq B$.

Definition 2.5. Suppose $A < M$ and $b \in M$ a single element. $b \perp A$ if $\text{cl}(bA) = bA$ and $d(b/A) = d(b)$.

Fact 2.6. Suppose $A < M$ and $b_1, b_2 \in M$ be single elements. If $b_1 \perp A$ and $b_2 \perp A$ then b_1 and b_2 are conjugate over A in M .

Proof. Suppose $b_1 \perp A$ and $b_2 \perp A$. We have $\text{cl}(b_1A) = b_1A$ by the definition. So, $\delta(b_1/A) = \delta(b_1) = 1$. This means that there are no edges between b_1 and A . By the same argument, there are no edges between b_2 and A . Hence, b_1A and b_2A are isomorphic over A and also $b_1A < M$ and $b_2A < M$. Therefore, the partial isomorphism between b_1A and b_2A over A can be extended to an automorphism of M by Fact 1.7. \square

Fact 2.7. If $M = \text{cl}(AD)$ for some finite $A < M$ and a basic orbit D over A then M is monodimensional.

Fact 2.8. If M is monodimensional then the automorphism group of M is a simple group.

3. HRUSHOVSKI'S BOUNDARY FUNCTIONS

Definition 3.1 ([7]). Let α be a positive real number. We define x_n, e_n, k_n, d_n for integers $n \geq 1$ by induction as follows: Put $x_1 = 2$ and $e_1 = 1$. Assume that x_n and e_n are defined. Let r_n be a smallest rational number r such that $r = k/d > \alpha$ with $d \leq e_n$ where k and d are positive integers. Let k_n and d_n be coprime positive integers with $k_n/d_n = r_n$. Finally, let $x_{n+1} = x_n + k_n$, and $e_{n+1} = e_n + d_n$.

Let $a_0 = (0, 0)$, and $a_n = (x_n, x_n - e_n\alpha)$ for $n \geq 1$. Let f be a function from \mathbb{R}^+ to \mathbb{R}^+ whose graph on interval $[x_n, x_{n+1}]$ with $n \geq 0$ is a line segment connecting a_n and a_{n+1} . We call f a *Hrushovski's boundary function associated to α* .

Fact 3.2 ([7]). *Let $D = B \otimes_A C$. Suppose B is normal to f , $x_n \leq |B| < x_{n+1}$ and $\delta(C/A) = x - e\alpha > 0$ with positive integers x, e . If $x/e \geq k_n/d_n$ then D is normal to f .*

Fact 3.3 ([7]). *Let $D = B \otimes_A C$. If $\delta(A) < \delta(B)$, $\delta(A) < \delta(C)$, and A, B, C are normal to f then D is normal to f .*

Fact 3.4 ([7]). *Let f be a Hrushovski's boundary function associated to α . Then f is strictly increasing and concave, and $(\mathbf{K}_f, <)$ has the free amalgamation property. Therefore, there is a generic structure of $(\mathbf{K}_f, <)$. Any one point structure is closed in any structure in \mathbf{K}_f . If α is rational then f is unbounded.*

In the rest of the paper, we fix $\alpha = 5/8$.

Note that $2 \cdot 8 - 3 \cdot 5 = 1$. Therefore, $2 - 3\alpha = 1/8$.

Proposition 3.5. (1) *Let $k \geq 0$ be an integer. Whenever $0 < x < 3 + 8(k+1)$ and $y/x > 5/8$ then $y/x \geq (2 + 5k)/(3 + 8k)$.*
 (2) *We refer to Definition 3.1. Suppose $e_n \geq 3$. Let l be a largest integer l' with $3 + 8l' \leq e_n$. Then $k_n = 2 + 5l$ and $d_n = 3 + 8l$.*

Proof. (2) follows from (1). So, we show (1).

First, note that for any integers u, v , $8v - 5u = 1$ if and only if $u = 3 + 8k$ and $v = 2 + 5k$ with an integer k . Also, $(2 + 5k)/(3 + 8k)$ is decreasing on k .

By inspection, whenever $0 < x < 3 + 8 = 11$ and $y/x > 5/8$ then $y/x \geq 2/3$.

Suppose $u = 3 + 8k$ and $v = 2 + 5k$ with $k \geq 1$. We have

$$\frac{v}{u} - \frac{5}{8} = \frac{1}{8u}.$$

Assume $u < x < u + 8$ and $y/x > 5/8$. Note that $x < 2u$ because $8 < u$. We have

$$\frac{y}{x} - \frac{5}{8} = \frac{8y - 5x}{8x} \geq \frac{2}{8x} > \frac{1}{8u}.$$

Therefore,

$$\frac{y}{x} > \frac{v}{u}.$$

We have (1). □

By this proposition, we have a following chart:

n	1	2	3	4	5	6	7	8	9
x_n	2	3	4	6	8	10	17	29	51
e_n	1	2	3	6	9	12	23	42	77
k_n	1	1	2	2	2	7	12	22	47
d_n	1	1	3	3	3	11	19	35	75

Also, for $n \geq 3$, we have

$$f(x_{n+1}) = f(x_n) + \frac{1}{8}.$$

The following are easy.

- Lemma 3.6.** (1) Let $C = A \otimes_p B$ where p is a single vertex and $A, B \in \mathbf{K}_f$. Then $C \in \mathbf{K}_f$.
(2) Any finite forest belongs to \mathbf{K}_f .
(3) Any cycle of length 6 or more belongs to \mathbf{K}_f .

Lemma 3.7. Let $B = A \otimes_{\{x,y\}} P$ where $P = x \cdots y$ is a path of length 3 or more. If the distance of x and y is 3 or more in A and $A \in \mathbf{K}_f$ then $B \in \mathbf{K}_f$.

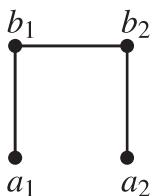
Lemma 3.8. Let $B = A \otimes_{\{x,y,z\}} P$ where

$$V(P) = \{x, y, z, x', y', z', w\} \text{ and } E(P) = \{xx', x'w, yy', y'w, zz', z'w\}.$$

If the pairwise distances among x, y, z in A are 2 or more and $A \in \mathbf{K}_f$ then $B \in \mathbf{K}_f$.

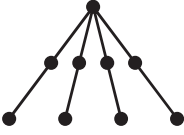
4. SPECIAL STRUCTURES

Let B be a graph with $V(B) = \{a_1, a_2, b_1, b_2\}$ and $E(B) = \{a_1b_1, a_2b_2, b_1b_2\}$ and let $A = \{a_1, a_2\}$. Then $A < B$ and $\delta(B/A) = 1/8$.



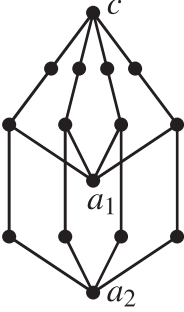
Suppose that B is a closed subset of M . Then b_1 and b_2 are basic over A because $1/8$ is the smallest positive possible dimension.

Let W_1 be the following graph:



Let F_1 be the set of leaves of W_1 . Then $F_1 <^- W_1$ and $\delta(W_1/F_1) = 0$.

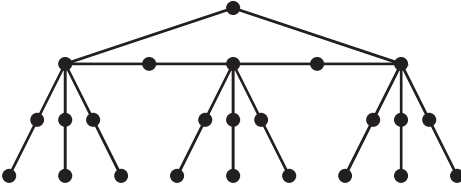
Make a free amalgam of 4 copies of B over A and attaching W_1 to it, we get:



The point c belongs to the closure of basic points over A .

Unfortunately, This structure does not belong to \mathbf{K}_f . But it turns out that any proper substructures belong to \mathbf{K}_f .

We can make a “wreath” W_3 with 3 copies of W_1 :

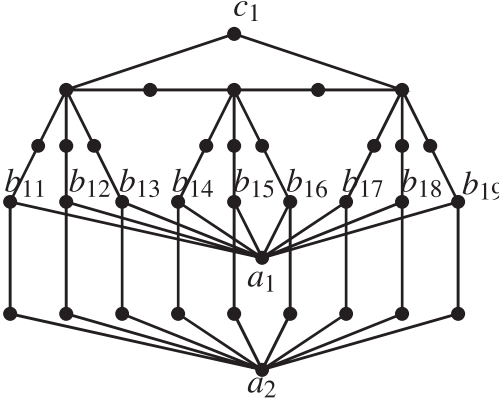


Let F be the leaves of W_3 . Then $F <^- W_3$ and $\delta(W_3/F) = 0$. A cycle with length 6 belongs to \mathbf{K}_f . Therefore, W_3 belong to \mathbf{K}_f .

Lemma 4.1. *Let*

$$C_1 = (B_1 \otimes_A B_2 \otimes_A \cdots \otimes_A B_9) \otimes_F W_3$$

where each B_i is isomorphic to B over A , $F = \{b_{11}, b_{12}, \dots, b_{19}\}$, with $b_{1i} \in B_i$ the isomorphic images of b_1 , and F is also the set of leaves of W_3 . Then $C_1 \in \mathbf{K}_f$. The following is a picture of C_1 :



Proof. Let S be the cycle of length 6 at the top including c_1 in C_1 . We can represent C_1 as

$$C_1 = H_1 \otimes_{AS} H_2 \otimes_{AS} H_3 \otimes_{AS} H_4 \otimes_{AS} H_5 \otimes_{AS} H_6 \otimes_{AS} H_7 \otimes_{AS} H_8 \otimes_{AS} H_9$$

where each H_i is $B_i p_i S$, p_i is adjacent to b_{1i} and to a single vertex in S . We have $H_i \in \mathbf{K}_f$ and $AS <^- H_i$ for each i .

We have to show that if $U \subseteq C_1$ then U is normal to f .

Case $U \cap A \neq A$. In this case, $U \cap AS < U \cap H_i$ for each i . Therefore, $U \in \mathbf{K}_f$ by the free amalgamation property.

Case $A \subseteq U$.

Subcase 1: $U = B_I \otimes_{F_I} U'$ where I is a subset of $\{1, 2, \dots, 9\}$, $B_I = \otimes_A \{B_i\}_{i \in I}$, $F_I = \{b_{1i}\}_{i \in I}$, and U' is a substructure of W_3 .

Let x be the number of points in $U' - F_I$ and e the number of edges in U' . To show that U is normal to f , we can assume that $e \leq 3|I|$ and $x < 2|I|$ by Lemmas 3.6, 3.7, and 3.8.

Suppose $|I| \leq 4$. Then $|B_I| = x_{2+|I|}$, and $e \leq 3|I| = e_{2+|I|}$. Since $x - e\alpha = \delta(U'/F_I) > 0$, we have $x/e > \alpha$. Hence, $x/e \geq k_{2+|I|}/d_{2+|I|}$. Therefore, U' is normal to f by Fact 3.3.

Suppose $|I| > 4$. We have $\delta(B_I) = 2 + |I|/8$, and $|B_I| = 2 + 2|I|$. $|B_I|$ is 12, 14, 16, 18, 20 for $|I| = 5, 6, 7, 8, 9$, respectively.

We also have $f(10) = 2 + 4/8$, $f(17) = 2 + 5/8$, $f(29) = 2 + 6/8$, and $f(51) = 2 + 7/8$.

Suppose $|I| = 5$. $|U| \leq 12 + 10 = 22$, and $\delta(U) \geq 2 + 5/8 + 1/8 = 2 + 6/8$. Since $|U| \leq 22 < 29$, U is normal to f .

Suppose $|I| = 6$. $|U| \leq 14 + 12 = 28$, and $\delta(U) \geq 2 + 6/8 + 1/8 = 2 + 7/8 > f(29)$. Since $|U| \leq 28 < 29$, U is normal to f .

Suppose $|I| = 7$. $|U| \leq 16 + 14 = 30$, and $\delta(U) \geq 2 + 7/8 + 1/8 = 2 + 1$. Since $|U| \leq 30 < 51$, and $f(51) = 2 + 7/8 < \delta(U)$, U is normal to f .

Suppose $|I| = 8$. $|U| \leq 18 + 16 = 34$, and $\delta(U) \geq 2 + 8/8 + 1/8 = 2 + 9/8$. Since $|U| \leq 30 < 51$, and $f(51) = 2 + 7/8 < \delta(U)$, U is normal to f .

Finally, suppose $|I| = 9$. $|U| \leq 20 + 18 = 38$, and $\delta(U) \geq 2 + 9/8$. Since $|U| \leq 38 < 51$, and $f(51) = 2 + 7/8 < \delta(U)$, U is normal to f .

Now, consider the general case. We can assume that $A \subseteq U$, and U is smooth over $AS \cap U$. Let I be the set of i such that $B_i \subseteq U \cap H_i$.

Let $U_I = \bigotimes_{AS \cap U} \{H_i \cap U\}_{i \in I}$, and $j \notin I$.

Put $U_j'' = U_I \otimes_{AS \cap U} H_j$. Then U_I and U_j'' are normal to f by Subcase 1. Put $U_j = U_I \otimes_{AS \cap U} (H_j \cap U)$. Then $|U_j| < |U_j''|$ and $\delta(U_j) \geq \delta(U_j'')$ because $\delta(H_j \cap U / AS \cap H_j \cap U) \geq \delta(H_j / AS \cap H_j)$. Hence, U_j is normal to f . Also, we have $U_I < U_j$. Therefore,

$$U = \bigotimes_{U_I} \{U_j\}_{j \notin I}$$

is normal to f by Fact 3.2. □

Lemma 4.2. *Let $C_2 = C_1 \otimes_{c_1} c_1 pc$ where $c_1 pc$ is a path of length 2. Then $C_2 \in \mathbf{K}_f$ and $Ac < C_2$.*

Proof. $C_2 \in \mathbf{K}_f$ because C_1 and $c_1 pc$ belong to \mathbf{K}_f and one point is always closed in any structure.

By inspection, for any $U \subseteq C_1$ with $A \subseteq U$ and $c_1 \in U$, we have $\delta(U/A) \geq 1$. Therefore, for any $U \subseteq C_2$ with $A \subseteq U$ and $c \in U$, we have $\delta(U/A) > 1$. Therefore, $\delta(U/Ac) = \delta(U/A) - 1 > 0$. Hence, $Ac < C_2$. □

Lemma 4.3. *Let $C_3 = \bigotimes_{Ac} \{C_{11}, C_{12}, C_{13}, C_{14}\}$ where C_{1j} are isomorphic to C_1 over Ac . Then $C_3 \in \mathbf{K}_f$, $A < C_3$, $Ac < C_3$.*

5. MONODIMENSIONALITY

We can prove the following theorem as in [5].

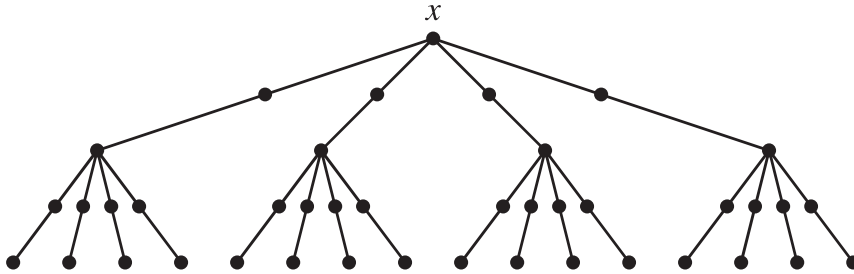
Theorem 5.1. *The generic structure M of \mathbf{K}_f is monodimensional. Therefore, the automorphism group of M is a simple group.*

Proof. Consider C_3 from 4.3. We can assume a_1 and a_2 are not connected and $A = \{a_1, a_2\} < M$. So, we can embed C_3 as a closed substructure of M . We can assume that $C_3 < M$. Isomorphic images of B over A in C_3 are also closed in M . Therefore, isomorphic images of b_1 are in a same basic orbit

over A , say D . Hence, $c \in \text{cl}(A, D)$. Since $cA < C_3$, cA is closed in M and thus $c \perp A$.

By Fact 2.6, we have $\{e \in M \mid e \perp A\} \subseteq \text{cl}(A, D)$.

Now, let $x \in M - A$. Let $X = \text{cl}(xA)$. Consider the following structure U :



Then $X < X \otimes_x U$. Embed this structure over X as a closed structure of M . So, we can assume $XU < M$. For each leaf y of U , $xy < U$. Therefore, $Ay < M$. Hence, $y \perp A$. On the other hand, by the structure of U , x is in the closure of the leaves of U . Thus, $x \in \text{cl}(\{e \in M \mid e \perp A\})$. Therefore, $x \in \text{cl}(A, D)$. \square

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